

Chapter 1

Partial restoration of chiral symmetry in hot and dense neutron matter

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We review efforts to describe the approach to chiral symmetry restoration in neutron matter from the low-energy realization of QCD, chiral effective field theory.

1. Introduction

The restoration of chiral symmetry in hot and dense hadronic matter and the associated observable signatures in relativistic heavy ion collisions were favorite research interests of Gerry Brown for two decades.^{1–5} Gerry attacked the problem with his characteristic directness, making an early prediction^{1,2} with Mannque Rho that nucleons and heavy mesons shed mass in the approach to chiral symmetry restoration according to a universal scaling law. At the time that one of the authors (JWH) became Gerry's graduate student at Stony Brook in the mid-2000's, numerous experimental investigations of the Brown-Rho scaling conjecture were underway.^{6–8} Despite the lack of clean experimental signatures^{4,5} for Brown-Rho scaling at finite density and temperature, understanding the effects of medium-modified meson masses and couplings on the nucleon-nucleon potential was nevertheless an inspiring topic for a PhD thesis.

One of Gerry's favorite words of wisdom for young graduate students was, "Build simple models and don't let anyone tell you it has to be more

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complicated.” The present contribution to this memorial volume will focus on a somewhat “more complicated” method to understand medium-modified nucleon-nucleon (NN) interactions and the approach to chiral symmetry restoration in hot and dense neutron matter. This framework, utilizing microscopic chiral effective field theory two- and three-body nuclear forces, in fact produces in-medium two-body interactions qualitatively similar to models incorporating Brown-Rho scaling.^{9–15} When employed in many-body perturbation theory to calculate the neutron matter thermodynamic equation of state, the coarse-resolution chiral potentials used in the present work show excellent convergence properties. Here again we are indebted to the work of Gerry Brown, who together with Tom Kuo, Scott Bogner and Achim Schwenk, pioneered the use of low-momentum nucleon-nucleon potentials¹⁶ derived from effective interaction theory and the renormalization group.

Chiral symmetry generically appears in quantum field theories with massless fermions. In the chiral limit of QCD the left- and right-handed quarks decouple and the Lagrangian is invariant under independent $SU(2)_{L,R}$ transformations in flavor space. In addition to the explicit breaking of chiral symmetry due to the small but nonzero bare quark mass arising from coupling to the Higgs field, the strong attraction between quark-antiquark pairs leads to the formation of a scalar quark condensate $\langle 0|\bar{q}q|0\rangle$ in the QCD vacuum that spontaneously breaks chiral symmetry. At the high temperatures and/or densities encountered in core-collapse supernovae or neutron star mergers, however, chiral symmetry may be restored.^{17,18}

At low densities the quark condensate in nuclear matter decreases linearly^{19,20} and proportional to the pion-nucleon sigma term $\sigma_{\pi N} = m_q \partial M_N / \partial m_q$, which encodes the small change in the nucleon mass M_N from the explicit breaking of chiral symmetry. Neglecting interaction contributions to the ground state energy, the linear term alone gives rise to chiral symmetry restoration in cold symmetric nuclear matter and pure neutron matter at a density $n \simeq 2.5n_0$, assuming a value of $\sigma_{\pi N} = 45 \text{ MeV}$,²¹ where $n_0 = 0.16 \text{ fm}^{-3}$. This is in the vicinity of the energy density at which chiral symmetry is restored at finite temperature and zero net baryon density from lattice QCD.²² Corrections to the leading density dependence have been obtained from the quark-mass dependence of interaction contributions to the nuclear matter ground state energy in one-boson-exchange models,^{23,24} in-medium chiral perturbation theory,^{25,26} and by employing high-precision chiral nuclear forces at next-to-next-to-next-to-leading order (N³LO).²⁷ In all of these studies, nuclear mean fields and correlations

arising from two- and three-body interactions consistently suppress chiral symmetry restoration beyond nuclear matter saturation density, relative to the noninteracting case.

At finite temperature, chiral perturbation theory was used to predict a phase transition to chiral symmetry restored matter at a critical temperature of $T_c \simeq 190$ MeV,²⁸ which is somewhat larger than the presently accepted value of $T_c = 155 \pm 10$ MeV from lattice QCD.^{29–31} In the present study we compute both the temperature and density dependence of the scalar quark condensate in neutron matter from high-precision two- and three-nucleon interactions. Previous finite-temperature calculations of the quark condensate in symmetric nuclear matter in the framework of in-medium chiral perturbation theory³² revealed that thermal effects wash out the interaction contributions that tend to delay chiral symmetry restoration at high densities. This leads to a nearly linear density dependence of the condensate for temperatures greater than $T = 50$ MeV. Hot proto-neutron stars born immediately after core-collapse supernovae or the hypermassive neutron stars that exist transiently after the merger of two neutron stars may therefore be more compelling candidate sites for quark-hadron phase transitions than cold neutron stars.

2. Neutron matter at finite temperature from many-body perturbation theory

Chiral effective field theory is the appropriate tool to study hadronic matter at the scales relevant in nuclear astrophysics (well below the chiral symmetry breaking scale of $\Lambda_\chi \simeq 1$ GeV). We start from a coarse-resolution chiral potential^{33,34} with a momentum-space cutoff of $\Lambda = 414$ MeV, which has been shown to exhibit good convergence properties^{33–37} in many-body perturbation theory calculations of infinite nuclear matter, comparable to low-momentum potentials constructed via renormalization group methods.^{16,38–40} In the present calculation the free energy per particle of pure neutron matter at finite temperature is computed in the imaginary-time Matsubara formalism. The perturbation series for the grand canonical potential Ω reads

$$\Omega(\mu, T) = \Omega_0(\mu, T) + \lambda \Omega_1(\mu, T) + \lambda^2 \Omega_2(\mu, T) + \cdots, \quad (1)$$

where λ is an arbitrary strength parameter, T is the temperature, and μ is the chemical potential. Eq. (1) can be reformulated in the canonical ensemble through the Kohn-Luttinger-Ward prescription.^{41,42} The result

is a rearrangement of the perturbation series:

$$F(\mu_0, T) = F_0(\mu_0, T) + \lambda \Omega_1(\mu_0, T) + \lambda^2 \left(\Omega_2(\mu_0, T) - \frac{1}{2} \frac{(\partial \Omega_1 / \partial \mu_0)^2}{\partial^2 \Omega_0 / \partial \mu_0^2} \right) + \dots, \quad (2)$$

where μ_0 is the chemical potential of the noninteracting system. There is a one-to-one correspondence between the nucleon density n and the effective chemical potential μ_0 through

$$n(\mu_0, T) = -\frac{\partial \Omega_0}{\partial \mu_0}, \quad (3)$$

where the noninteracting grand canonical potential has the well known form

$$\Omega_0(\mu_0, T) = -\frac{1}{\beta} \frac{1}{\pi^2} \int_0^\infty dp p^2 \log(1 + e^{-\beta(e(p) - \mu_0)}) \quad (4)$$

and $e(p) = p^2/2M_N$ is the single-particle energy. The first term in the expansion of the free energy is related to Ω_0 via $F_0(\mu_0, T) = \Omega_0(\mu_0, T) + n\mu_0$.

The first- and second-order perturbative contributions to the free energy of neutron matter are shown diagrammatically in Fig. 1. The wavy lines represent the sum of the free-space NN interaction and an in-medium NN potential constructed from the N2LO chiral three-body force by summing one leg over the filled Fermi sea of noninteracting neutrons. This approximation can be improved upon by including three-body forces at N3LO⁴³ and by treating three-body forces explicitly at higher orders in perturbation theory.^{44,45} In the present case, the effective interaction depends on the density and temperature of neutron matter. The explicit expressions for the diagrams shown in Fig. 1 are given by

$$\Omega_1(\mu_0, T) = \frac{1}{2} \sum_{\sigma_1, \sigma_2} \int \frac{d^3 k_1 d^3 k_2}{(2\pi)^6} n_{k_1} n_{k_2} \langle 12 | (\bar{V}_{NN} + \bar{V}_{NN}^{med}/3) | 12 \rangle, \quad (5)$$

$$\begin{aligned} \Omega_2^n(\mu_0, T) = & -\frac{1}{8} \prod_{i=1}^4 \left(\sum_{\sigma_i} \int \frac{d^3 k_i}{(2\pi)^3} \right) (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_3 - \vec{k}_4) \\ & \times \frac{n_{k_1} n_{k_2} \bar{n}_{k_3} \bar{n}_{k_4} - \bar{n}_{k_1} \bar{n}_{k_2} n_{k_3} n_{k_4}}{e_3 + e_4 - e_1 - e_2} |\langle 12 | (\bar{V}_{NN} + \bar{V}_{NN}^{med}) | 34 \rangle|^2, \end{aligned} \quad (6)$$

where n_k is the Fermi distribution function, $\bar{n}_k = 1 - n_k$, and \bar{V} is the antisymmetrized potential. We have written only the expression for the “normal” second-order contribution Ω_2^n and neglected the anomalous contribution, which was shown³⁵ to have a negligible effect when combined with the derivative term in Eq. (2). The free energy of neutron matter has been computed previously³⁶ employing the n3lo414 chiral potential. At low

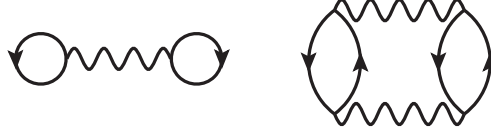


Fig. 1. First- and second-order diagrammatic contributions to the free energy of pure neutron matter from n3lo414 chiral two- and three-nucleon forces. The wavy line represents a density- and temperature-dependent NN interaction derived from the chiral three-body force.

densities and high temperatures, the results were found to agree well with the model-independent virial expansion, and in the vicinity of nuclear matter saturation density the symmetry energy $E_{sym} = 32.5 \text{ MeV}$ and slope $L = 53.8 \text{ MeV}$ are consistent with empirical constraints.⁴⁶

From the Hellmann-Feynman theorem^{20,32}

$$\langle \psi(\alpha) | \frac{d}{d\alpha} H(\alpha) | \psi(\alpha) \rangle = \frac{d}{d\alpha} E(\alpha), \quad (7)$$

the chiral condensate in finite-density matter is related to the vacuum value by choosing $\alpha = m_q$:

$$\langle \psi | \bar{q}q | \psi \rangle - \langle 0 | \bar{q}q | 0 \rangle = \frac{1}{2} \frac{d}{dm_q} [M_N n + \mathcal{E}], \quad (8)$$

where $|\psi\rangle$ is the neutron matter ground state and \mathcal{E} is the energy density. Since we do not have direct access to the quark-mass dependence of the neutron matter ground state energy, we employ the Gell-mann–Oakes–Renner relation at leading order

$$m_\pi^2 f_\pi^2 \simeq -2\bar{m}_q \langle 0 | \bar{q}q | 0 \rangle, \quad (9)$$

where \bar{m}_q is the average light-quark mass and $f_\pi = 92.4 \text{ MeV}$ is the pion decay constant, to write the chiral condensate in terms of the pion-mass dependence of the energy per particle \bar{E} :

$$\frac{\langle \bar{q}q \rangle_n}{\langle \bar{q}q \rangle_0} = 1 - \frac{n}{f_\pi^2} \left[\frac{\sigma_{\pi N}}{m_\pi^2} + \frac{d\bar{E}}{dm_\pi^2} \right]. \quad (10)$$

This expression can be extended to finite temperature³² by replacing the energy per particle in Eq. (10) with the free energy per particle \bar{F}

$$\frac{\langle \bar{q}q \rangle_{n,T}}{\langle \bar{q}q \rangle_{0,0}} = 1 - \frac{n}{f_\pi^2} \frac{\partial \bar{F}(n, T)}{\partial m_\pi^2}, \quad (11)$$

and absorbing the nucleon mass and chemical potential μ_0 into \bar{F} .

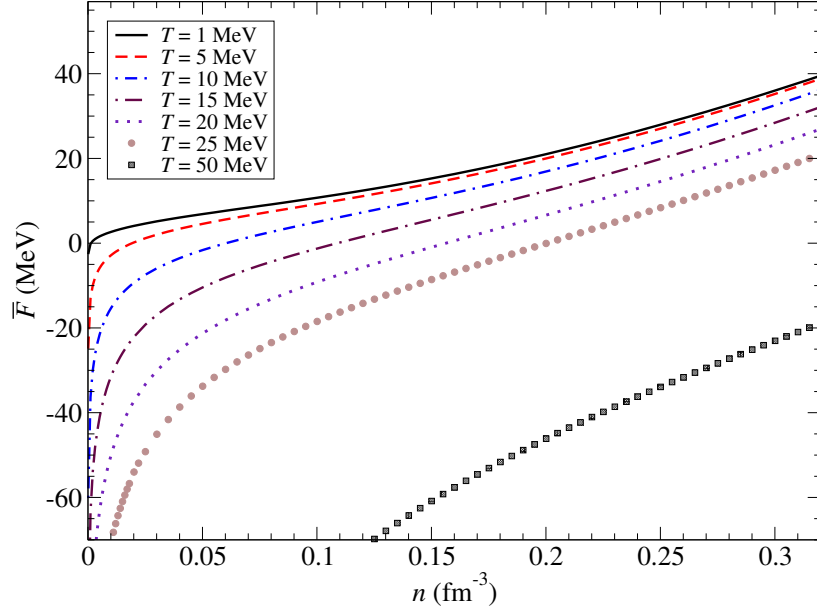


Fig. 2. Free energy per particle \bar{F} of pure neutron matter as a function of density for different isotherms. Results are obtained at second order in many-body perturbation theory employing the coarse-resolution chiral potential n3lo414 described in the text.

3. Results

We begin by plotting in Fig. 2 the neutron matter thermodynamic equation of state (the free energy per particle as a function of temperature and density) obtained from the n3lo414 chiral two- and three-body potentials. The results are in good agreement with previous studies,^{36,37} which employed the same two- and three-body potentials but included also self-consistent Hartree-Fock single-particle energies in the second-order diagrams. As we will show later, second-order perturbative contributions to the neutron matter equation of state and the chiral condensate are relatively small. The inclusion of in-medium nucleon propagators will therefore be postponed to future work and higher-order contributions to the nucleon single-particle potential will be addressed.

In Fig. 3 we show the ratio of the chiral condensate in neutron matter at finite temperature T to the chiral condensate in vacuum as a function of the nucleon number density n . In computing the numerical derivatives of the different contributions to the free energy density with respect to the pion

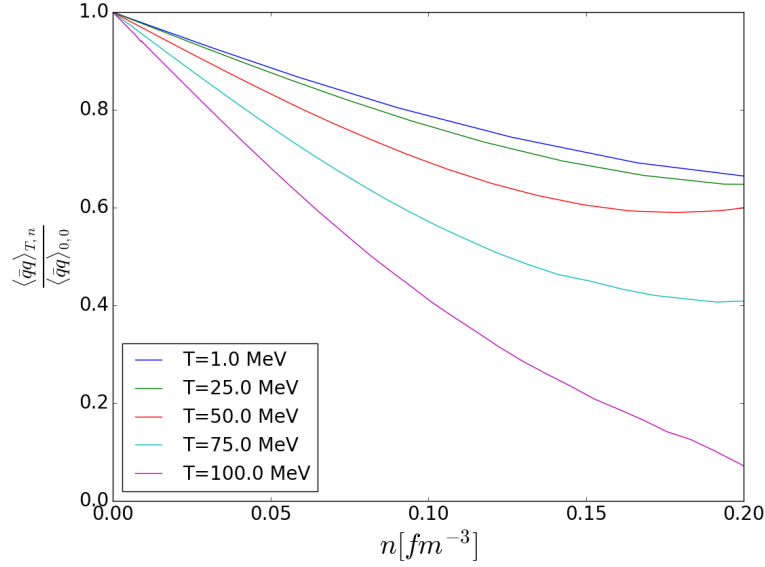


Fig. 3. Ratio of the chiral condensate in pure neutron matter to that in the vacuum as a function of density and temperature from the pion-mass dependence of the free energy computed in many-body perturbation theory with coarse-resolution chiral two- and three-body potentials.

mass, we chose to vary the pion mass by 5%, 2%, and 1%. We found only very small differences in the values of the numerical derivatives, and in the figure the results from a 1% change in the pion mass are shown. We observe that at zero temperature the density dependence of the chiral condensate falls within the uncertainty band obtained in Ref.,²⁷ which included as well three-body forces at N3LO in the chiral power counting.

Temperatures up to 100 MeV are considered in the present work. In the highest temperature regime, the presence of thermal pions enhance the trend toward chiral symmetry restoration,^{28,32,47} leading to a qualitative difference in the condensate ratio especially at low density. In the present work, we focus on nuclear interaction contributions to the medium dependence of the chiral condensate and will include effects due to thermal pions in future work. We neglect also the quark-mass dependence of the axial coupling strength g_A , pion decay constant f_π , and short-range low-energy constants in the 2N and 3N sectors. For zero-temperature neutron matter, these effects were estimated²⁷ to contribute on the order of 25% to the theoretical uncertainty in the chiral condensate. More detailed discussions can

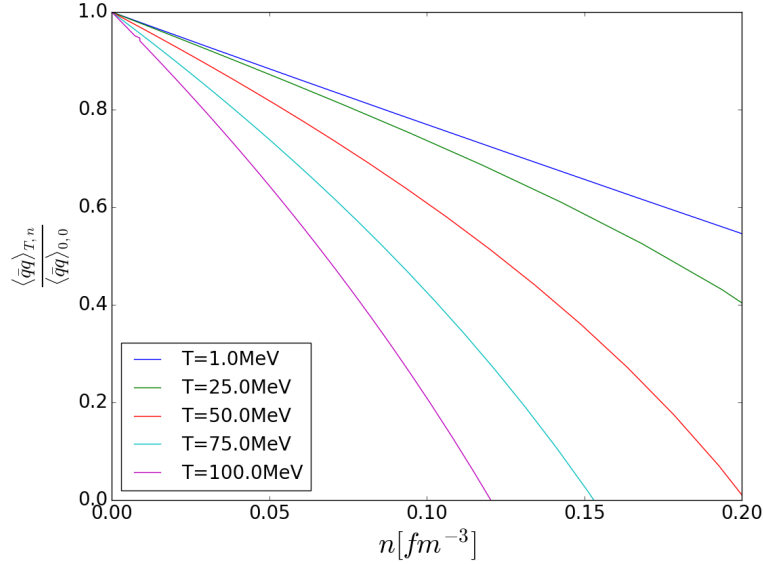


Fig. 4. Same as in Fig. 3, except that only the noninteracting contribution to the medium dependence of the chiral condensate is included.

be found in the literature.^{48,49}

As found in previous chiral effective field theory calculations^{26,27} of the pion-mass dependence of the neutron matter ground state energy at zero temperature, we observe that interaction contributions result in a relatively small change in the leading linear decrease of the chiral condensate with increasing density. In particular, repulsive interactions delay the approach to chiral symmetry restoration and increase with nucleon density. At $n = 0.2 \text{ fm}^{-3}$ nuclear interactions increase the chiral condensate by about 15%. This is a larger effect, though qualitatively similar, to what has been observed employing one-boson-exchange models of the nucleon-nucleon interaction.²³

Increasing the temperature, on the other hand, is highly effective at promoting chiral symmetry restoration in neutron matter. This comes predominantly from the noninteracting contributions to the pion mass dependence of the neutron matter free energy, as shown in Fig. 4. Increasing the quark mass enhances both the value of the nucleon mass and the kinetic energy contribution to the neutron matter free energy. In Figs. 5 and 6 we plot the first-order, $\Delta^{(1)} \frac{\langle \bar{q}q \rangle_{T,n}}{\langle \bar{q}q \rangle_{0,0}}$, and second-order, $\Delta^{(2)} \frac{\langle \bar{q}q \rangle_{T,n}}{\langle \bar{q}q \rangle_{0,0}}$, perturbative contributions to the chiral condensate ratio from two- and three-nucleon chiral

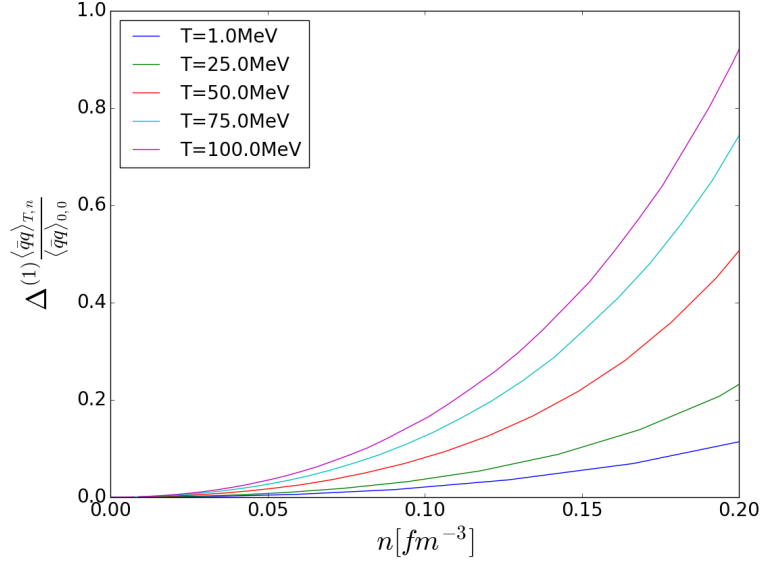


Fig. 5. Change in the chiral condensate ratio as a function of temperature and density including the first-order perturbative contribution to the free energy of neutron matter.

forces. For all densities and temperatures considered in the present work, both the first- and second-order contributions tend to delay the restoration of chiral symmetry for increasing density and temperature. This feature was observed already in Ref.²⁷ for neutron matter at zero temperature, and we note that it holds with increasing temperature. The leading Hartree-Fock contribution is consistently about three times larger in magnitude than the second-order correction. For temperatures up to $T = 50$ MeV and densities below $n = 0.2 \text{ fm}^{-3}$, the second-order interaction contribution to the scalar quark condensate gives less than a 10% correction. This is in agreement with previous studies of the neutron matter equation of state where the use of coarse-resolution chiral potentials strongly reduced the effect of the second-order bubble diagram. Uncertainties associated with the pion-nucleon sigma term and low-energy constants in two- and three-nucleon forces are therefore expected to dominate the theoretical errors.²⁷

4. Summary

Many-body perturbation theory with coarse-resolution chiral potentials now allows for the systematic study of infinite nuclear matter properties

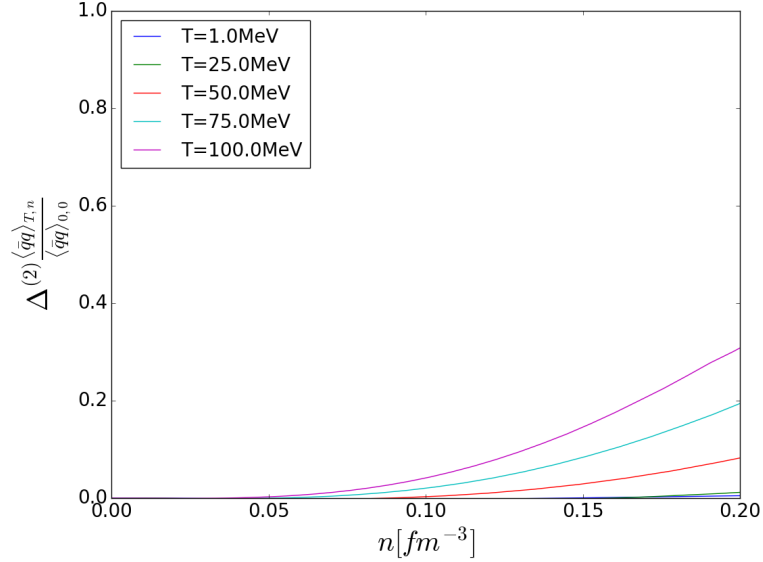


Fig. 6. Same as in Fig. 5, except that only the second-order perturbative contribution to the free energy has been included.

with reduced theoretical uncertainties. In the present work we have calculated the scalar quark condensate in neutron matter at finite temperature employing a recently developed chiral nuclear potential with a momentum-space cutoff of $\Lambda = 414$ MeV. We considered separately the pion-mass dependence of noninteracting contributions to the free energy of neutron matter as well as interaction effects from two- and three-body nuclear forces. For all densities and temperatures, the noninteracting contributions are dominant and lead to a decrease of the chiral condensate with increasing density and temperature. In the absence of nuclear interactions, hot neutron matter around nuclear matter saturation density should already exhibit a transition to a chiral symmetry restored phase. However, interaction effects generically delay chiral symmetry restoration and increase in magnitude with both the density and temperature.

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